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TURBULENT HEAT TRANSFER AND TEMPERATURE FLUCTUATIONS IN A FIELD WITH UNIFORM VELOCITY AND TEMPERATURE GRADIENTS

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Abstract—Equations are derived from the Navier–Stokes and energy equations for the correlations between velocities and temperatures at two points in a homogeneous turbulent field. Although uniform mean velocity and temperature gradients are present, in the field, the turbulence decays with time. Solutions are obtained by converting the equations to spectral form by taking their Fourier transforms and assuming that the turbulence is sufficiently weak for triple correlations to be negligible in comparison with double correlations. Spectra of the turbulent heat transfer and of the mean square temperature fluctuation are calculated as a function of dimensionless velocity gradient. The ratio of eddy diffusivity for heat transfer to that for momentum transfer is also obtained. It is shown that the eddy diffusivity ratio approaches one at high velocity gradients regardless of the value of Prandtl number. However, its rate of approach to 1 is greatest for Prandtl numbers on the order of 1.

NOMENCLATURE

a, transverse velocity gradient, dU_1/dx_2;
a^*, dimensionless transverse velocity gradient, $(t - t_o)dU_1/dx_2$;
b, transverse temperature gradient, dT/dx_2;
J_o, constant that depends on initial conditions;
P, P', arbitrary points in turbulent field;
Pr, Prandtl number, ν/α;
p, instantaneous pressure;
r, distance from point P to P';
$\mathbf{r}$, distance vector from point P to P';
r_k, component of $\mathbf{r}$;
T, average temperature;
$\tilde{T}$, instantaneous temperature;
T_t, transfer term for temperature fluctuations obtained by integrating the quantity $\kappa_1 \partial\delta/\partial\kappa_2$ in equation (34) over all directions in wave number space;
t, time;
t_o, reference time;
U_k, an averaged velocity component;
$\tilde{U}_k$, instantaneous velocity component;
u_k, fluctuating part of velocity component defined by (4);
x_k, space co-ordinate.

Greek symbols

α, thermal diffusivity;
Γ_2, spectrum function of $\overline{\tau u_2}$ defined by (44);
Γ_2^*, dimensionless spectrum function of $\overline{\tau u_2}$, $\nu^2(t - t_o)\Gamma_2/(J_o b)$;
γ_j, Fourier transform of $\overline{\tau u_j'}$ defined by (20);
γ_2^*, dimensionless Fourier transforms of $\overline{\tau u_j'}$, $\nu\gamma_j/(J_o b)$;
γ_i', Fourier transform of $\overline{u_i \tau'}$ defined by (21);
Δ, spectrum function of $\overline{\tau^2}$ defined by (45);
Δ^*, dimensionless spectrum function of $\overline{\tau^2}$, $\nu^2\Delta/(J_o b^2)$;
δ, Fourier transform of $\overline{\tau\tau'}$ defined by (22);
δ^*, dimensionless Fourier transform of $\overline{\tau\tau'}$, $\nu\delta/[J_o(t - t_o)b^2]$;
δ_{ij}, equals 1 for $i = j$ and equals 0 for $i \neq j$;
ϵ, eddy diffusivity for momentum transfer defined by (51);
ϵ^*, dimensionless eddy diffusivity for momentum transfer, $\nu^{5/2}(t - t_o)^{3/2}\epsilon/J_o$;

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ϵ_h, eddy diffusivity for heat transfer defined by (50);
ϵ^*, dimensionless eddy diffusivity for heat transfer, $\nu^{5/2}(t - t_0)^{3/2}\epsilon_h/J_0$;
ζ, Fourier transform of $\overline{p\tau'}$ defined by (24);
ζ', Fourier transform of $\overline{\tau p'}$ defined by (25);
θ, spherical co-ordinate in wave number space;
κ, wave number;
κ^*, dimensionless wave number, $\nu^{1/2}(t - t_0)^{1/2}\kappa$;
$\boldsymbol{\kappa}$, wave number vector;
κ_i, component of wave number vector;
ν, kinematic viscosity;
ρ, density;
τ, fluctuating part of temperature defined by (3);
ϕ, spherical co-ordinate in wave number space.

Subscripts
i, j, k, subscripts that have the values 1, 2, or 3 and designate co-ordinate directions.

Superscripts
$'$, referring to point P';
$^-$, overbar designates average value.

INTRODUCTION

TURBULENT flow and heat transfer in passages and boundary layers are usually analysed by using a phenomenological approach. That is, assumptions are introduced into the analysis to relate the turbulent shear stress and turbulent heat transfer to the mean flow. Summaries of these analyses are given in references [1]–[3]. This approach is very useful and makes it possible to generalize large quantities of experimental data. In fact, it appears to be the only feasible way, at present, of analysing the complex flows occurring in boundary layers and passages.

Although the phenomenological analyses are very useful, we can obtain a great deal more insight into the turbulent processes by using a statistical approach based on the equations of motion and energy. These studies should help to put our phenomenological analyses on a sounder basis. Because of the complexity of turbulence it is necessary to limit ourselves, at least at the beginning, to simple models, when studying it from a fundamental standpoint. Thus, Corrsin [4] and Dunn and Reid [5] studied heat transfer in isotropic turbulence with a uniform mean temperature gradient. (The term "isotropic" as used here indicates that the statistical properties of the turbulence are independent of direction.) Recently, the author analysed anisotropic turbulence in the presence of a uniform vertical temperature gradient and body force [6].

In the present paper we study the heat transfer and temperature fluctuations in a statistically homogeneous turbulent field with uniform transverse velocity and temperature gradients. The heat transfer in this case is somewhat similar to that in passages and boundary layers, although the turbulence in those cases would be inhomogeneous because of the presence of walls. The turbulence is assumed to be weak enough for the triple correlations occurring in the analysis to be negligible in comparison with double correlations. That is, a first-order analysis is considered, and the present results evidently represent a limiting case. Higher order corrections could be obtained by the method used in [7] and [8]. As shown in [9], even though triple correlations are neglected, the turbulent field considered here has all the features commonly associated with turbulence, including a transfer of energy between eddies of various sizes. Also, because of the presence of the production and velocity-gradient transfer terms in the correlation equations, the triple correlation terms, which do not contain velocity gradient, may be relatively less important than in the case of no velocity gradient. Thus the results may apply at moderate as well as low turbulence Reynolds number.

The fluid properties are assumed constant, so that the turbulent velocity field is independent of the temperature field. Thus the results for turbulence with a uniform velocity gradient from [9] can be used for obtaining the turbulent heat transfer and temperature fluctuations. It is shown in [9] that a homogeneous turbulent field with a uniform velocity gradient decays with time. Although energy is fed into the turbulence from the mean velocity gradient, the production of turbulence is never great enough to offset the dissipation. The fluctuating temperature field and the turbulent heat transfer will also change with time.

Because of the decay of the turbulence with time it will be necessary to produce it initially by some means, for instance, by passing a stream through a grid. Then various distances downstream from the grid will correspond to various times of decay. Approximately uniform transverse velocity and temperature gradients in the stream could be produced by passing the flow through parallel channels before passing it through the grid. The temperature and velocity of the fluid in each channel would be adjusted to produce the desired velocity and temperature gradients across the stream emerging from the channels. Because of the higher velocities through some parts of the grid it might be necessary to vary the thickness of the wires in the grid to produce an approximately homogeneous turbulence. Heating of the grid would not be necessary because, as will be seen, temperature fluctuations can arise from the interaction of the turbulence and the mean temperature gradient.

In order to proceed with the analysis, it is necessary to construct, from the equations of motion and energy, equations involving correlations between fluctuating quantities at two points in the fluid. These equations will be obtained in the next section.

BASIC EQUATIONS

Consider first the energy or heat transfer equation. This equation can be written for two arbitrary points in the fluid P and P' as

$$\frac{\partial \tilde{T}}{\partial t} + \frac{\partial(\tilde{u}_k \tilde{T})}{\partial x_k} = \alpha \frac{\partial^2 \tilde{T}}{\partial x_k \partial x_k} \tag{1}$$

$$\frac{\partial \tilde{T}'}{\partial t} + \frac{\partial(\tilde{u}_k' \tilde{T}')}{\partial x_k'} = \alpha \frac{\partial^2 \tilde{T}'}{\partial x_k' \partial x_k'} \tag{2}$$

where a repeated subscript in a term indicates a summation of terms, with the subscripts successively taking on the values 1, 2, and 3. The quantity $\tilde{T}$ is the instantaneous temperature, u_k is a velocity component, x_k is a space coordinate, t is the time, and α is the thermal diffusivity. The instantaneous quantities in (1) and (2) can be divided into mean and fluctuating components. Thus, in (1) we let

$$\tilde{T} = T + \tau \tag{3}$$

and

$$\tilde{u}_k = U_k + u_k. \tag{4}$$

Then (1) becomes

$$\frac{\partial T}{\partial t} + \frac{\partial \tau}{\partial t} + U_k \frac{\partial T}{\partial x_k} + U_k \frac{\partial \tau}{\partial x_k} + U_k \frac{\partial T}{\partial x_k} + \frac{\partial(\tau u_k)}{\partial x_k} = \alpha \left(\frac{\partial^2 T}{\partial x_k \partial x_k} + \frac{\partial^2 \tau}{\partial x_k \partial x_k} \right). \tag{5}$$

Averaging (5) over time or over a large number of identical systems (ensemble average) gives

$$\frac{\partial T}{\partial t} + U_k \frac{\partial T}{\partial x_k} + \frac{\partial \overline{\tau u_k}}{\partial x_k} = \alpha \frac{\partial^2 T}{\partial x_k \partial x_k} \tag{6}$$

where the overbar indicates an average value. In obtaining (6), use was made of the fact that $\tau = \overline{u_k} = 0$. Subtraction of (6) from (5) gives

$$\frac{\partial \tau}{\partial t} + U_k \frac{\partial \tau}{\partial x_k} + u_k \frac{\partial T}{\partial x_k} + \frac{\partial(\tau u_k)}{\partial x_k} - \frac{\partial \overline{\tau u_k}}{\partial x_k} = \alpha \frac{\partial^2 \tau}{\partial x_k \partial x_k}. \tag{7}$$

Equation (7) applies at a point P in the fluid. The corresponding equation for a point P' is

$$\frac{\partial \tau'}{\partial t} + U_k' \frac{\partial \tau'}{\partial x_k'} + u_k' \frac{\partial T'}{\partial x_k'} + \frac{\partial(\tau' u_k')}{\partial x_k'} - \frac{\partial \overline{\tau' u_k'}}{\partial x_k'} = \alpha \frac{\partial^2 \tau'}{\partial x_k' \partial x_k'}. \tag{8}$$

By a similar procedure it is shown in [9] from the Navier–Stokes equations that

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial U_i}{\partial x_k} + U_k \frac{\partial u_i}{\partial x_k} + \frac{\partial}{\partial x_k}(u_i u_k) - \frac{\partial}{\partial x_k} \overline{u_i u_k} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} \tag{9}$$

and

$$\frac{\partial u_j'}{\partial t} + u_k' \frac{\partial U_j'}{\partial x_k'} + U_k' \frac{\partial u_j'}{\partial x_k'} + \frac{\partial}{\partial x_k'}(u_j'u_k') - \frac{\partial}{\partial x_k'}(\overline{u_j'u_k'}) = -\frac{1}{\rho}\frac{\partial p'}{\partial x_j'} + \nu \frac{\partial^2 u_j'}{\partial x_k' \partial x_k'}. \quad (10)$$

Note that (9) and (10) each represents three equations since i and j can take on the values 1, 2, or 3. To obtain an equation for $\overline{\tau u_j'}$, multiply (7) by u_j' and (10) by τ, add, and take averages. This gives

$$\frac{\partial \overline{\tau u_j'}}{\partial t} + U_k \frac{\partial \overline{\tau u_j'}}{\partial x_k} + \overline{u_k u_j'}\frac{\partial T}{\partial x_k} + \frac{\partial}{\partial x_k}\overline{\tau u_k u_j'} + \overline{\tau u_k'}\frac{\partial U_j'}{\partial x_k'} + U_k' \frac{\partial \overline{\tau u_j'}}{\partial x_k'} + \frac{\partial}{\partial x_k'}\overline{\tau u_j' \tau u_k'} = -\frac{1}{\rho}\frac{\partial \overline{\tau p'}}{\partial x_j'} + \nu \frac{\partial^2 \overline{\tau u_j'}}{\partial x_k' \partial x_k'} + \alpha \frac{\partial^2 \overline{\tau u_j'}}{\partial x_k \partial x_k}. \quad (11)$$

In obtaining (11), fluctuating quantities were placed inside the spacial derivative signs by using the fact that quantities at one point are independent of the position of the other point. If **r** is the vector extending from **x** to **x**$'$, $(\partial/\partial x_k')_{x_k} = \partial/\partial r_k$ and $(\partial/\partial x_k)_{x_k'} = -\partial/\partial r_k$ for homogeneous turbulence. If, in addition, the temperature and velocity gradients are uniform and exist only in the transverse direction x_2,

$$(U_k' - U_k)\frac{\partial}{\partial r_k}\overline{\tau u_j'} = \frac{\partial U_1}{\partial x_2} r_2 \frac{\partial}{\partial r_1}\overline{\tau u_j'} \quad (12)$$

and (11) becomes

$$\frac{\partial}{\partial t}\overline{\tau u_j'} + \frac{\partial U_1}{\partial x_2} r_2 \frac{\partial}{\partial r_1}\overline{\tau u_j'} + \overline{u_2 u_j'}\frac{\partial T}{\partial x_2} + \overline{\tau u_2'}\,\delta_{1j}\frac{\partial U_1}{\partial x_2} + \frac{\partial}{\partial r_k}(\overline{\tau u_j' u_k'} - \overline{\tau u_k u_j'}) = -\frac{1}{\rho}\frac{\partial}{\partial r_j}\overline{\tau p'} + (\alpha + \nu)\frac{\partial^2 \overline{\tau u_j'}}{\partial r_k \partial r_k}. \quad (13)$$

The symbol $\delta_{1j} = 1$ for $j = 1$ and 0 for $j \neq 1$.

By using a similar procedure, one obtains from (8) and (9),

$$\frac{\partial}{\partial t}\overline{u_i \tau'} + \overline{u_2 \tau'}\,\delta_{i1}\frac{\partial U_1}{\partial x_2} + \overline{u_i u_2'}\frac{\partial T}{\partial x_2} + \frac{\partial U_1}{\partial x_2} r_2 \frac{\partial}{\partial r_1}\overline{u_i \tau'} + \frac{\partial}{\partial r_k}(\overline{u_i u_k' \tau'} - \overline{u_i u_k \tau'}) = \frac{1}{\rho}\frac{\partial}{\partial r_i}\overline{p\tau'} + (\alpha + \nu)\frac{\partial^2 \overline{u_i \tau'}}{\partial r_k \partial r_k} \quad (14)$$

and from (7) and (8),

$$\frac{\partial}{\partial t}\overline{\tau\tau'} + \frac{\partial U_1}{\partial x_2} r_2 \frac{\partial}{\partial r_1}\overline{\tau\tau'} + \frac{\partial T}{\partial x_2}(\overline{u_2\tau'} + \overline{\tau u_2'}) + \frac{\partial}{\partial r_k}(\overline{\tau\tau' u_k'} - \overline{u_k \tau\tau'}) = 2\alpha \frac{\partial^2 \overline{\tau\tau'}}{\partial r_k \partial r_k}. \quad (15)$$

To obtain expressions for the correlations containing pressure fluctuations, differentiate equation (9) with respect to x_i and (10) with respect to x_j' and apply the continuity equation

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial u_j'}{\partial x_j'} = 0. \quad (16)$$

This gives, from equation (10)

$$\frac{1}{\rho}\frac{\partial^2 p'}{\partial x_j' \partial x_j'} = -2\frac{\partial u_k'}{\partial x_j'}\frac{\partial U_j'}{\partial x_k'} - \frac{\partial^2 (u_j' u_k')}{\partial x_j' \partial x_k'} + \frac{\partial^2 \overline{u_j' u_k'}}{\partial x_j' \partial x_k'}. \quad (17)$$

Multiplying (17) by τ, averaging, and introducing the variable $r_j = x_j' - x_j$ gives

$$\frac{1}{\rho}\frac{\partial^2 \overline{\tau p'}}{\partial r_j \partial r_j} = -2\frac{\partial U_1}{\partial x_2}\frac{\partial \overline{\tau u_2'}}{\partial r_1} - \frac{\partial^2 \overline{\tau u_j' u_k'}}{\partial r_j \partial r_k}. \quad (18)$$

Where the velocity gradient is again assumed to be uniform and in the x_2-direction. Similarly,

$$\frac{1}{\rho}\frac{\partial^2 \overline{p\tau'}}{\partial r_i \partial r_i} = 2\frac{\partial U_1}{\partial x_2}\frac{\partial \overline{u_2\tau'}}{\partial r_1} - \frac{\partial^2 \overline{u_i u_k \tau'}}{\partial r_i \partial r_k}. \quad (19)$$

In order to simplify (13), (14), (15), (18), and (19), and because of the physical significance of spectral quantities, we introduce the following three-dimensional Fourier transforms [10]:

$$\overline{\tau u_j'}(\mathbf{r}) = \int_{-\infty}^{\infty} \gamma_j(\boldsymbol{\varkappa}) e^{i\boldsymbol{\varkappa}\cdot\mathbf{r}}\, d\boldsymbol{\varkappa} \quad (20)$$

$$\overline{u_i\tau'}(\mathbf{r}) = \int_{-\infty}^{\infty} \gamma_i'(\boldsymbol{\varkappa}) e^{i\boldsymbol{\varkappa}\cdot\mathbf{r}}\, d\boldsymbol{\varkappa} \quad (21)$$

$$\overline{\tau\tau'}(\mathbf{r}) = \int_{-\infty}^{\infty} \delta(\boldsymbol{\varkappa}) e^{i\boldsymbol{\varkappa}\cdot\mathbf{r}}\, d\boldsymbol{\varkappa} \quad (22)$$

$$\overline{u_i u_j'}(\mathbf{r}) = \int_{-\infty}^{\infty} \phi_{ij}(\boldsymbol{\varkappa}) e^{i\boldsymbol{\varkappa}\cdot\mathbf{r}}\, d\boldsymbol{\varkappa} \quad (23)$$

$$\overline{p\tau'}(\mathbf{r}) = \int_{-\infty}^{\infty} \zeta(\boldsymbol{\varkappa}) e^{i\boldsymbol{\varkappa}\cdot\mathbf{r}}\, d\boldsymbol{\varkappa} \quad (24)$$

$$\overline{\tau p'}(\mathbf{r}) = \int_{-\infty}^{\infty} \zeta'(\boldsymbol{\varkappa}) e^{i\boldsymbol{\varkappa}\cdot\mathbf{r}}\, d\boldsymbol{\varkappa} \quad (25)$$

where $\boldsymbol{\varkappa}$ is known as the wave number vector and $d\boldsymbol{\varkappa} = d\kappa_1 d\kappa_2 d\kappa_3$. The magnitude of $\boldsymbol{\varkappa}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. The dot product $\boldsymbol{\varkappa}\cdot\mathbf{r}$ could also be written as $\kappa_k r_k$. Thus, differentiation of, say, $\overline{\tau u_j'}$ with respect to r_k will multiply its Fourier transform γ_j by $i\kappa_k$. Inversely, multiplication of a quantity by r_k will differentiate its Fourier transform by κ_k and multiply it by i. Taking the Fourier transforms of (13), (14), (15), (18), and (19) and assuming that the turbulence is weak enough for triple correlations to be negligible in comparison with double correlations result in

$$\frac{\partial \gamma_j}{\partial t} - \frac{\partial U_1}{\partial x_2}\kappa_1 \frac{\partial \gamma_j}{\partial \kappa_2} + \phi_{2j}\frac{\partial T}{\partial x_2} + \delta_{1j}\gamma_2 \frac{\partial U_1}{\partial x_2} = -\frac{1}{\rho} i\kappa_j \zeta' - (\alpha + \nu)\kappa^2 \gamma_j \quad (26)$$

$$\frac{\partial \gamma_i'}{\partial t} - \frac{\partial U_1}{\partial x_2}\kappa_1 \frac{\partial \gamma_i'}{\partial \kappa_2} + \delta_{i1}\gamma_2' \frac{\partial U_1}{\partial x_2} + \phi_{i2}\frac{\partial T}{\partial x_2} = \frac{1}{\rho} i\kappa_i \zeta - (\alpha + \nu)\kappa^2 \gamma_i'. \quad (27)$$

$$\frac{\partial \delta}{\partial t} - \frac{\partial U_1}{\partial x_2}\kappa_1 \frac{\partial \delta}{\partial \kappa_2} + \frac{\partial T}{\partial x_2}(\gamma_2 + \gamma_2') = -2\alpha\kappa^2\delta \quad (28)$$

$$\frac{1}{\rho} i\kappa_i \zeta = 2\frac{\kappa_1\kappa_i}{\kappa^2}\gamma_2' \frac{\partial U_1}{\partial x_2} \quad (29)$$

and

$$-\frac{1}{\rho} i\kappa_j \zeta' = 2\frac{\kappa_1\kappa_j}{\kappa^2}\gamma_2 \frac{\partial U_1}{\partial x_2} \quad (30)$$

where use is made of the fact that $\kappa_k\kappa_k = \kappa_1^2 + \kappa_2^2 + \kappa_3^2 = \kappa^2$. Substituting (29) and (30) into the right-hand sides of (26) and (27), letting $i = j = 2$, and comparing the resulting equations, shows that $\gamma_2 = \gamma_2'$ for all times if they are equal at an initial time. Here it will be assumed that the temperature fluctuations are initially zero, so that the above relation will hold. If

$$\partial U_1/\partial x_2 \equiv a \quad (31)$$

and

$$\partial T/\partial x_2 \equiv b \quad (32)$$

we finally obtain

$$\frac{\partial \gamma_2}{\partial t} - a\kappa_1 \frac{\partial \gamma_2}{\partial \kappa_2} = -b\phi_{22} + \left[2a\frac{\kappa_1\kappa_2}{\kappa^2} - \left(\frac{1}{Pr} + 1\right)\nu\kappa^2\right]\gamma_2 \quad (33)$$

and

$$\frac{\partial \delta}{\partial t} - a\kappa_1 \frac{\partial \delta}{\partial \kappa_2} = -2b\gamma_2 - 2\alpha\kappa^2\delta. \quad (34)$$

SOLUTION OF SPECTRAL EQUATIONS

In order to obtain solutions of (33) and (34) it will be assumed, as in [9], that the turbulence is initially isotropic, although it is not, of course, isotropic at later times.

The expression for ϕ_{22} in (33), which is the Fourier transform of $\overline{u_2 u_2'}$, has already been obtained in [9]. This expression is

$$\phi_{22} = \frac{J_o\{\kappa_1^2 + [\kappa_2 + a\kappa_1(t - t_o)]^2 + \kappa_3^2\}^2(\kappa_1^2 + \kappa_3^2)}{12\pi^2\kappa^4} \cdot \exp\left\{-2\nu(t - t_o)\left[\kappa^2 + \frac{1}{3}\kappa_1^2 a^2 (t - t_o)^2 + a\kappa_1\kappa_2(t - t_o)\right]\right\} \quad (35)$$

where J_o and t_o are constants that depend on initial conditions. For a Prandtl number of 1 the solution of (33) is

$$\kappa^2 \exp\left\{2\nu(t-t_o)\left[\kappa^2 + a\kappa_1\kappa_2(t-t_o) + \frac{1}{3}a^2\kappa_1^2(t-t_o)^2\right]\right\}\gamma^2 = \frac{J_o\{\kappa_1^2 + [\kappa_2 + a\kappa_1(t-t_o)]^2 + \kappa_3^2\}^2(\kappa_1^2+\kappa_3^2)^{1/2}}{12\pi^2 a\kappa_1} \cdot b\tan^{-1}\frac{\kappa_2}{(\kappa_1^2+\kappa_3^2)^{1/2}} + f[\kappa_1, \kappa_2 + a\kappa_1(t-t_o), \kappa_3] \tag{36}$$

where f is a function of integration. The method of solution is given in [11]. In order to evaluate f, it is assumed that the temperature fluctuations are zero for $t = t_o$. Thus substituting $\gamma_2 = 0$ for $t = t_o$ in (36),

$$f(\kappa_1, \kappa_2, \kappa_3) = -\frac{J_o(\kappa_1^2+\kappa_2^2+\kappa_3^2)^2(\kappa_1^2+\kappa_3^2)^{1/2}}{12\pi^2 a\kappa_1} b\tan^{-1}\frac{\kappa_2}{(\kappa_1^2+\kappa_3^2)^{1/2}} \tag{37}$$

or

$$f[\kappa_1, \kappa_2 + a\kappa_1(t-t_o), \kappa_3] = -\frac{J_o\{\kappa_1^2 + [\kappa_2 + a\kappa_1(t-t_o)]^2 + \kappa_3^2\}^2(\kappa_1^2+\kappa_3^2)^{1/2}}{12\pi^2 a\kappa_1} \cdot b\tan^{-1}\frac{\kappa_2 + a\kappa_1(t-t_o)}{(\kappa_1^2+\kappa_3^2)^{1/2}}. \tag{38}$$

Substitution of (38) in (36) gives, for the Fourier transform of $\overline{\tau u_2'}$ for a Prandtl number of 1,

$$\gamma_2 = \frac{J_o\{\kappa_1^2 + [\kappa_2 + a\kappa_1(t-t_o)]^2 + \kappa_3^2\}^2(\kappa_1^2+\kappa_3^2)^{1/2}}{12\pi^2 a\kappa_1\kappa^2} \cdot b\exp\{-2\nu(t-t_o)[\kappa^2 + a\kappa_1\kappa_2(t-t_o) + \tfrac{1}{3}a^2\kappa_1^2(t-t_o)^2]\} \cdot \left[\tan^{-1}\frac{\kappa_2}{(\kappa_1^2+\kappa_3^2)^{1/2}} - \tan^{-1}\frac{\kappa_2 + a\kappa_1(t-t_o)}{(\kappa_1^2+\kappa_3^2)^{1/2}}\right]. \tag{39}$$

For $Pr \neq 1$, the solution for γ_2 can be written as

$$\gamma_2 = -\frac{b}{a\kappa_1\kappa^2}\frac{J_o(\kappa_1^2+\kappa_3^2)}{12\pi^2}\{\kappa_1^2 + [\kappa_2 + a\kappa_1(t-t_o)]^2 + \kappa_3^2\}^2 \cdot \exp\left\{\frac{[(1/Pr)-1]\,\nu\kappa_2}{a\kappa_1}\left(\kappa_1^2 + \frac{\kappa_2^2}{3} + \kappa_3^2\right) - 2\nu(t-t_o)[\kappa^2 + a\kappa_1\kappa_2(t-t_o) + \tfrac{1}{3}a^2\kappa_1^2(t-t_o)^2]\right\} \cdot \int_{\kappa_2}^{\kappa_2 + a\kappa_1(t-t_o)} \frac{1}{\kappa_1^2 + \xi^2 + \kappa_3^2} \exp\left\{-\left[\frac{[(1/Pr)-1]\,\nu}{a\kappa_1}\xi\left(\kappa_1^2 + \frac{\xi^2}{3} + \kappa_3^2\right)\right]\right\} d\xi. \tag{39a}$$

The expression for the Fourier transform of $\overline{\tau\tau'}$ for a Prandtl number of 1 is obtained by solution of (34):

$$\delta = \frac{J_o\{\kappa_1^2 + [\kappa_2 + a\kappa_1(t-t_o)]^2 + \kappa_3^2\}^2 b^2}{12\pi^2 a^2\kappa_1^2} \exp\{-2\nu(t-t_o)[\kappa^2 + a\kappa_1\kappa_2(t-t_o) + \tfrac{1}{3}a^2\kappa_1^2(t-t_o)^2]\} \left[\tan^{-1}\frac{\kappa_2}{(\kappa_1^2+\kappa_3^2)^{1/2}} - \tan^{-1}\frac{\kappa_2 + a\kappa_1(t-t_o)}{(\kappa_1^2+\kappa_3^2)^{1/2}}\right]^2 \tag{40}$$

where δ was set equal to zero for $t = t_o$.

The spectral quantities γ_2 and δ are functions of the components of the wave number vector $\boldsymbol{\kappa}$ as well as of its magnitude. It is somewhat easier to interpret quantities that are functions only of the magnitude κ. We can obtain such quantities in the usual way by integrating γ_2 and δ over all directions in wave number space. Thus, define a quantity Γ_2 by the equation

$$\Gamma_2(\kappa) = \int_0^A \gamma_2(\boldsymbol{\kappa})\, dA(\boldsymbol{\kappa}) \tag{41}$$

where A is the area of the surface of a sphere of radius κ. Then, since

$$\overline{\tau u_2} = \int_0^\infty \Gamma_2\, d\kappa \tag{42}$$

[let $r = 0$ in (20)], $\Gamma_2 \, d\kappa$ gives the contribution from wave number band $d\kappa$ to $\overline{\tau u_2}$. Thus a plot of Γ_2 against κ shows how contributions to $\overline{\tau u_2}$ are distributed among the various wave numbers or eddy sizes.

Equations (39) and (40) can be written in terms of spherical co-ordinates by setting

$$\kappa_1 = \kappa \cos\phi \sin\theta, \quad \kappa_2 = \kappa \sin\phi \sin\theta, \quad \kappa_3 = \kappa \cos\theta. \tag{43}$$

Then (41) becomes

$$\Gamma_2(\kappa) = \int_0^{\pi} \int_0^{2\pi} \gamma_2(\kappa, \phi, \theta)\kappa^2 \sin\theta \, d\phi \, d\theta. \tag{44}$$

A similar equation for δ integrated over all directions in wave number space is

$$\Delta(\kappa) = \int_0^{\pi} \int_0^{2\pi} \delta(\kappa, \phi, \theta)\kappa^2 \sin\theta \, d\phi \, d\theta. \tag{45}$$

Letting $r = 0$ in (22),

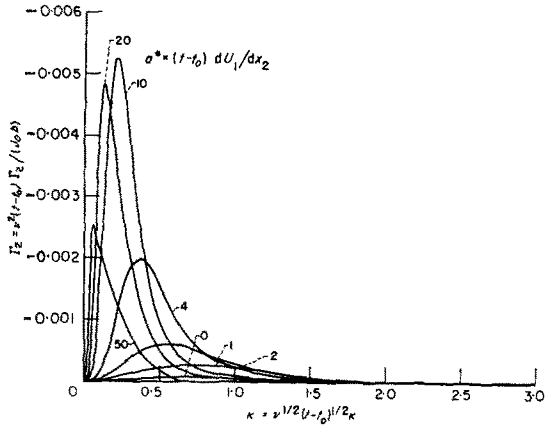
$$\overline{\tau^2} = \int_0^{\infty} \Delta \, d\kappa \tag{46}$$

so that, as in the case of Γ_2, $\Delta \, d\kappa$ gives contributions from the wave number band $d\kappa$ to $\overline{\tau^2}$.

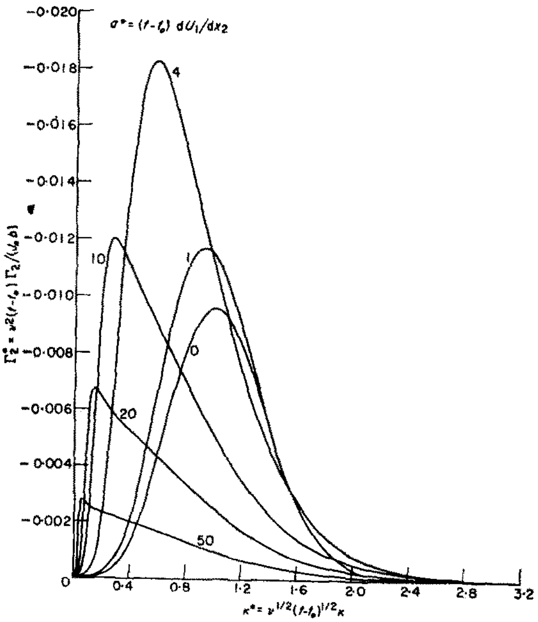
RESULTS AND DISCUSSION

Discussion of computed spectra

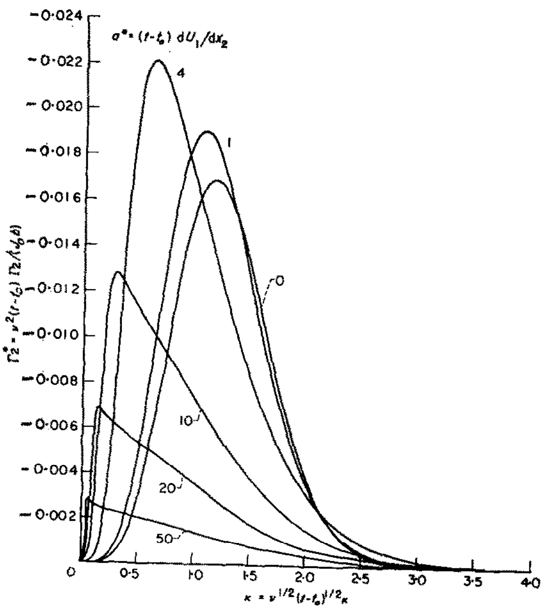
Equations (39), (40), (44), and (45) can be converted to dimensionless form by introducing the variables γ_2^*, δ^*, κ^*, a^*, Γ_2^*, and Δ^*. (These quantities are defined in the Nomenclature.) Calculated spectra of $\overline{\tau u_2}$ and $\overline{\tau^2}$ for various values of dimensionless velocity gradient a^* are plotted in Figs. 1 and 2. The integrations in (44), (45), and (39a) were carried out numerically.



(b)



(a)



(c)

FIG. 1. Dimensionless spectra of $\overline{\tau u_2}$ for uniform transverse velocity and temperature gradients. (a) Prandtl number = 1. (b) Prandtl number = 0·01. (c) Prandtl number = 10.

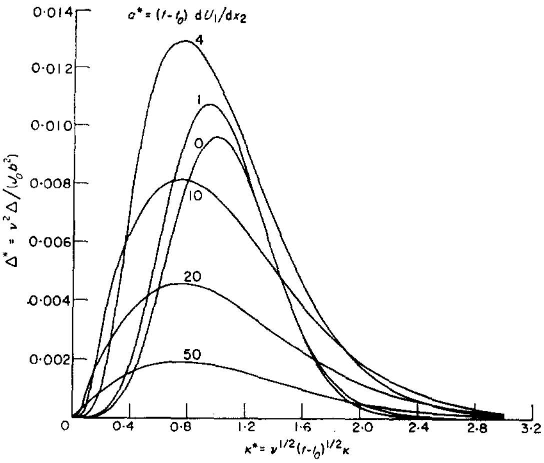


FIG. 2. Dimensionless spectra of $\overline{\tau^2}$ for uniform transverse velocity and temperature gradients. Prandtl number = 1.

When plotted using the similarity variables shown, the curves for zero velocity gradient do not change with time, so that comparison of the various curves indicates how the velocity gradient will alter the spectrum. Thus the curves in Figs. 1 and 2 that lie above those for $a^* = 0$ indicate that for those cases $\overline{\tau u_2}$ or $\overline{\tau^2}$ at a particular time is greater than it would be for no velocity gradient. The turbulence itself is, of course, decaying with time. Fig. 1 shows the effect of Prandtl number on the spectrum of $\overline{\tau u_2}$. As Prandtl number increases, the peaks of the spectra move toward the higher wave number region, the change being greater at the lower values of a^*. High wave numbers correspond to small eddies, inasmuch as the wave number represents the reciprocal of an eddy size (or wave length).

For zero velocity gradient the results are the same as those obtained by Dunn and Reid [5]. As the velocity gradient increases, the peaks of the spectra of $\overline{\tau u_2}$ move to lower wave numbers because the spectrum of the production term $b\phi_{22}$ in (33) moves to the left (see Fig. 5, reference [9]). Since the production term in the equation for the spectrum of $\overline{\tau^2}$, (34), is proportional to γ_2, the peaks of the spectra of $\overline{\tau^2}$ also move to lower wave numbers.

The spectra change from approximately symmetric curves to curves having more gradual slopes on the high wave number sides as a^* increases. The changes in shape of the spectra are apparently caused by a transfer of acitivity from low wave numbers to high wave numbers or from big eddies to small ones. This transfer is generally associated with triple correlations [7], but in the present case, where triple correlations are neglected, it is associated with the velocity gradient. Thus we can interpret the second terms in (33) and (34) as transfer terms. In order to do this, note that $r_2\,\partial\overline{\tau u_2'}/\partial r_1$ in (13) is related to $\kappa_1\,\partial\gamma_2/\partial\kappa_2$ in (33) by

$$r_2\frac{\partial\overline{\tau u_2'}}{\partial r_1} = -\int_{-\infty}^{\infty}\kappa_1\frac{\partial\gamma_2}{\partial\kappa_2}\,e^{i\boldsymbol{\kappa}\cdot\mathbf{r}}\,d\boldsymbol{\kappa}. \tag{47}$$

For $r = 0$, this becomes

$$\int_{-\infty}^{\infty}\kappa_1\frac{\partial\gamma_2}{\partial\kappa_2}\,d\boldsymbol{\kappa} = 0. \tag{48}$$

Similarly, in (34)

$$\int_{-\infty}^{\infty}\kappa_1\frac{\partial\delta}{\partial\kappa_2}\,d\boldsymbol{\kappa} = 0. \tag{49}$$

Thus these terms give zero total contribution to $\partial\overline{\tau u_2}/\partial t$ or to $\partial\overline{\tau^2}/\partial t$. However, they can alter the distribution in wave number space of contributions to $\partial\overline{\tau u_2}/\partial t$ or $\partial\overline{\tau^2}/\partial t$, and thus can be interpreted as transfer terms. A similar term in the equation for $\overline{u_i u_j'}$ was obtained in [9].

The expressions for the transfer terms in (33) and (34) can be integrated over all directions in wave number space by using equations similar to (44) and (45) in order to obtain quantities that are functions only of κ and dU_1/dx_2. A plot of the integrated transfer term corresponding to $\overline{\tau^2}$ is given in dimensionless form for a Prandtl number of 1 in Fig. 3. This term corresponds to the second term in (34) with the exception that it has not been multiplied by a. The total area enclosed by each curve is zero, in agreement with (49). The curves are predominately negative at low wave numbers and positive at higher ones, so that, in general contributions to $\overline{\tau^2}$ are transferred from low wave numbers to high ones. In this way the higher wave number portions of the spectra of $\overline{\tau^2}$ in Fig. 2 are excited by the transfer of activity into those regions, so that the shapes of the spectra are altered. This effect is similar to

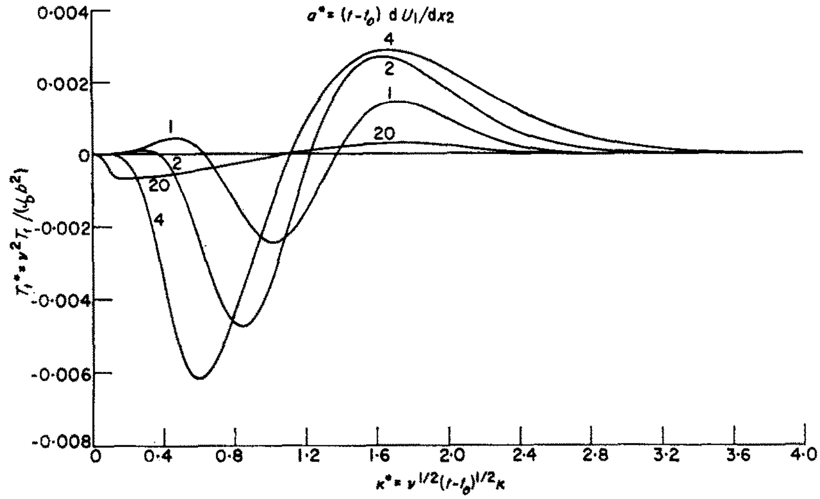


FIG. 3. Dimensionless spectra of transfer term due to mean velocity gradient in spectral equation for $\overline{\tau^2}$. Prandtl number = 1.

that due to triple correlations [7]. In the present case a natural explanation of the effect is that the transfer to higher wave numbers is due to the stretching of the vortex lines in the turbulence by the velocity gradient. The velocity gradient should also be able to compress some of the vortex lines, particularly at low velocity gradients where the orientation of the vortex lines would tend to be random. This might explain the small amount of reverse transfer at low wave numbers and low velocity gradients in Fig. 3.

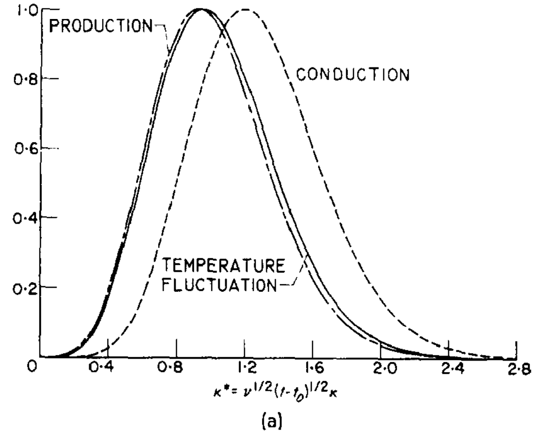
Production, temperature fluctuation, and conduction regions

By analogy with the equation for turbulent energy in [9], one can interpret the third term in (34) as being responsible for the production of temperature fluctuations by the action of the mean temperature gradient on the turbulent heat transfer $\overline{\tau u_2}$. In the corresponding production term in the turbulent energy equation [9] the mean velocity gradient does work on the turbulent shear stress. The last term in (34) is the conduction or dissipation term and tends to destroy the temperature fluctuations by conducting heat away from regions of high local temperature. This action is similar to the action of viscosity on the velocity fluctuations.

The production and conduction terms in (34) can be integrated over all directions in wave number space by substituting Γ_2 and Δ for γ_2 and δ respectively in those terms. These terms, together with the spectrum of $\overline{\tau^2}$ are plotted in normalized form in Figs. 4a, b for two values of a^* and a Prandtl number of 1. For the low dimensionless velocity gradient the production, temperature fluctuation, and conduction regions are but slightly separated. On the other hand, for the high velocity gradient ($a^* = 50$), the production takes place mostly in the low wave number or big eddy region and the conductive attenuation occurs in the high wave number region. The conductive attenuation occurs mostly in the high wave number region because conduction effects tend to "smear out" the small-scale temperature fluctuations more readily than the large ones.

We can summarize the history of the temperature fluctuations at high velocity gradients somewhat as follows: the temperature fluctuations are produced by the mean temperature gradient mainly in the big eddy region. This temperature fluctuation activity or "energy" is transferred from the big temperature eddies to smaller ones by the action of the velocity gradient. Finally the temperature "energy" is dissipated by conduction effects in the small eddy region.

The separation at high velocity gradients of the three regions shown in Fig. 4b is similar to the



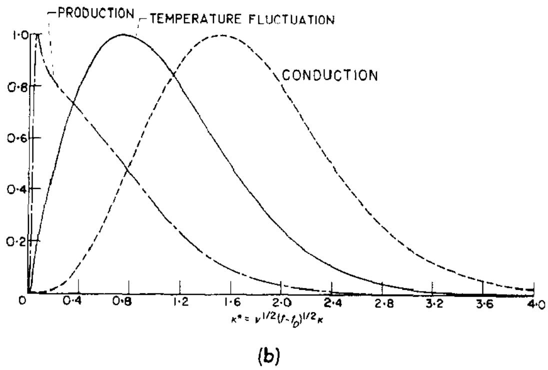


FIG. 4. Comparison of production, temperature fluctuation and conduction spectra from spectral equation for $\overline{\tau u_2}$. Prandtl number = 1. Curves are normalized to same height.
(a) $a^* = (t - t_0)\, dU_1/dx_2 = 1$.
(b) $a^* = (t - t_0)\, dU_1/dx_2 = 50$.

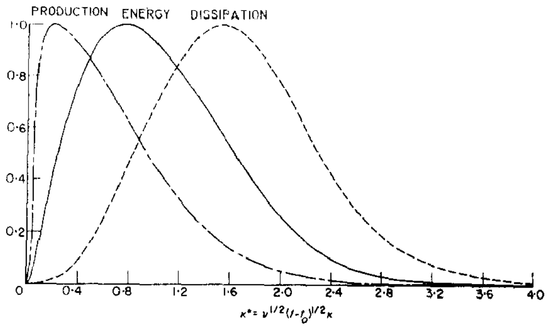


FIG. 5. Comparison of production, energy, and dissipation spectra from spectral equation for $\overline{u_i u_i}$. [$a^* = (t - t_0)\, dU_1/dx_2 = 50$].

separation of the production, energy-containing, and dissipation regions associated with the turbulent energy $\overline{u_i u_i}/2$. For comparison, a plot of these regions for an a^* of 50 is given in Fig. 5. These curves were obtained from equations given in [9].

Temperature–velocity correlation coefficient

The temperature–velocity correlation coefficient as introduced by Corrsin [4], is defined as $\overline{\tau u_2}/(\overline{\tau^2}\,\overline{u_2^2})^{1/2}$. For perfect correlation between τ and u_2, this coefficient will have a value of 1. The coefficient can be calculated by measuring the areas under the spectrum curves in Figs. 1 and 2 and in Fig. 5 of [9]. A plot of the temperature–velocity correlation coefficient against dimensionless velocity gradient is given for a Prandtl number of 1 in Fig. 6. For zero velocity gradient, perfect correlation between the temperature and velocity fluctuations is indicated. It should be mentioned that this result applies only to a Prandtl number of 1. The Prandtl number dependence of the coefficient for zero velocity gradient is given by equation (78) of reference [5]. As the velocity gradient increases, Fig. 6 indicates that the correlation between the temperature and velocity is partially destroyed. At a value of a^* of 50 the correlation coefficient has decreased to about 0·5.

Ratio of eddy diffusivities for heat transfer to momentum transfer

The eddy diffusivities for heat transfer and for momentum transfer are defined as

$$\epsilon_h = \frac{k\,\overline{\tau u_2}}{dT/dx_2} \tag{50}$$

and

$$\epsilon = \frac{k\,\overline{u_1 u_2}}{dU_1/dx_2}. \tag{51}$$

The eddy diffusivity ratio ϵ_h/ϵ is of considerable importance in the phenomenological theories of turbulent heat transfer and is usually assumed to be one [1, Section E]. In fact that assumption gives the best agreement between analysis and experiment, except, possibly at low Prandtl or Peclet numbers [12], [13]. A dimensionless eddy diffusivity for heat transfer $\nu^{5/2}(t - t_o)^{3/2}\epsilon_h/J_o$ can be obtained from the areas under the curves

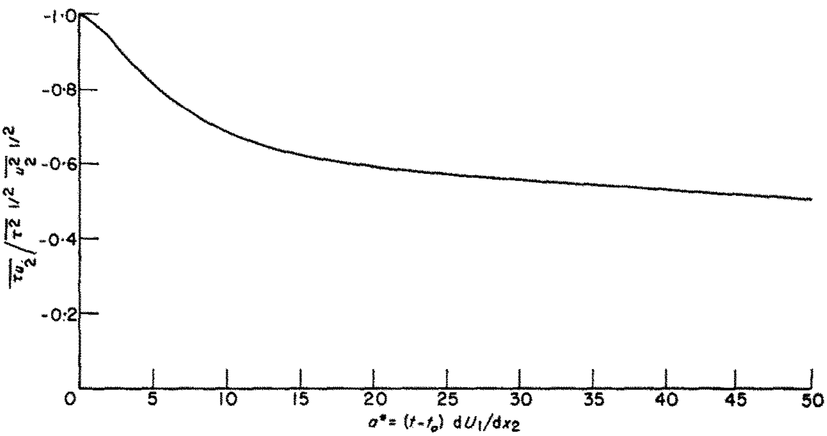


FIG. 6. Variation of temperature-velocity correlation coefficient with dimensionless velocity gradient. Prandtl number = 1.

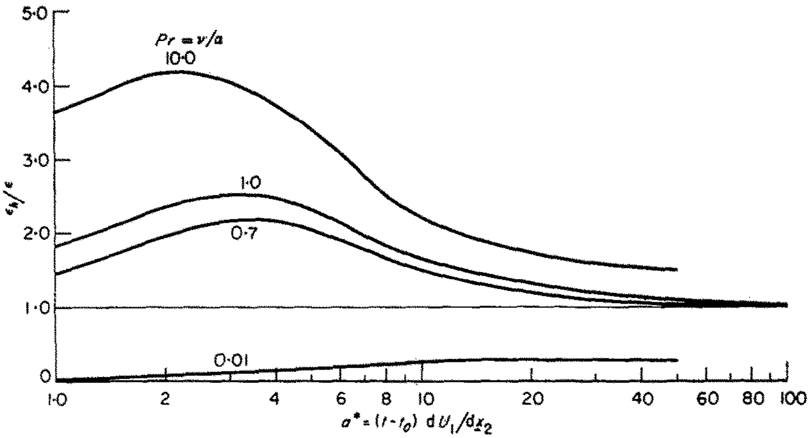


FIG. 7. Variation of ratio of eddy diffusivity for heat transfer to that for momentum transfer with dimensionless velocity gradient.

in Fig. 1. A similar dimensionless eddy diffusivity for momentum transfer is given in Fig. 9 of reference [9]. The ratio ϵ_h/ϵ is plotted in Figs. 7 and 8. Fig. 8 is included inasmuch as the eddy diffusivity ratio for $a^* = 0$ is not given in Fig. 7. This case corresponds to isotropic turbulence and can be calculated from the results in [5] and [9]. For small velocity gradients ϵ_h/ϵ is greater than 1 except for the low Prandtl number. However, as the velocity gradient increases, ϵ_h/ϵ ultimately decreases and approaches 1 at large velocity gradients. This is shown on a spectral basis in Fig. 9, where the dimensionless spectra of ϵ_h and ϵ for a Prandtl number of 1 are compared. As the velocity gradient increases, the spectrum curves of ϵ_h and ϵ approach each other rapidly in the high wave number or small eddy region and somewhat more slowly in the low wave number region.

The approach to 1 of ϵ_h/ϵ as the velocity gradient increases, occurs at all Prandtl numbers. This can be seen by inspection of (39a) which indicates that for large values of the velocity gradient a, the effect of Prandtl number on γ_2 and thus on ϵ_h is negligible. However, the effect of Prandtl number is much greater at low values of Pr than at higher ones. This is because the

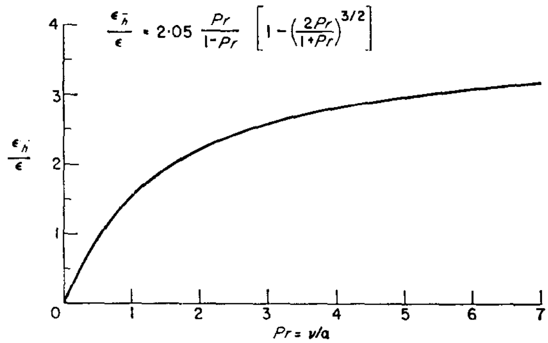


FIG. 8. Variation of ϵ_h/ϵ with Prandtl number for isotropic turbulence with velocity gradient = 0.

terms in (39a) which contain Prandtl number vary much more rapidly with low values of that quantity than with high ones.

Fig. 7 indicates that as the velocity gradient increases, the approach of ϵ_h/ϵ to 1 is most rapid for Prandtl numbers on the order of one and least rapid for very low Prandtl numbers.

It is of interest to compare the various terms in the differential equations for γ_2/b and ϕ_{12}/a at high values of a. The quantities γ_2/b and ϕ_{12}/a will, when integrated over wave number space, give ϵ_h and ϵ. Equation (33) can be written in terms of γ_2/b as

$$\frac{\partial(\gamma_2/b)}{\partial t} - a\kappa_1\frac{\partial(\gamma_2/b)}{\partial\kappa_2} = -\phi_{22} + 2a\frac{\kappa_1\kappa_2}{\kappa^2}\left(\frac{\gamma_2}{b}\right) - \left(\frac{1}{Pr}+1\right)\nu\kappa^2\left(\frac{\gamma_2}{b}\right). \tag{52}$$

From [9]

$$\frac{\partial(\phi_{12}/a)}{\partial t} - a\kappa_1\frac{\partial(\phi_{12}/a)}{\partial\kappa_2} = -\phi_{22} + 2\frac{a\kappa_1\kappa_2}{\kappa^2}\left(\frac{\phi_{12}}{a}\right) + 2\frac{\kappa_1^2}{\kappa^2}\phi_{22} - 2\nu\kappa^2\left(\frac{\phi_{12}}{a}\right). \tag{53}$$

These equations for γ_2/b and for ϕ_{12}/a are the same except for the last term in (52) and the last two terms in (53). It appears, however, from the forms of the equations that these terms should not be important for high values of a. The next to the last term in (53) arises from the pressure fluctuations.

Although (52) and (53) are similar for large values of a, the initial conditions for γ_2/b and ϕ_{12}/a are different, the initial form for ϕ_{12} being given by equation (43) in reference [9], whereas γ_2 is initially zero. However, a numerical check indicates that γ_2/b and ϕ_{12}/a, as well as the integrated values ϵ_h and ϵ, are essentially equal

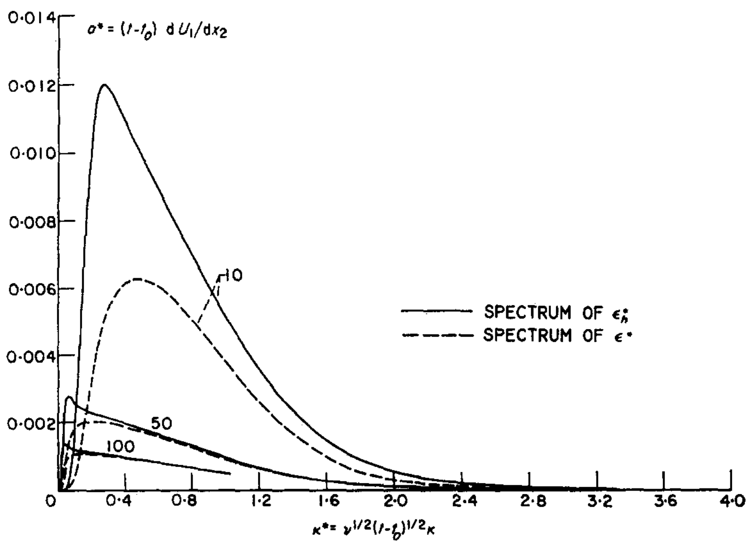


FIG. 9. Comparison of spectra of ϵ_h with those of ϵ. Prandtl number = 1.

for large values of a^*. This suggests that the initial conditions have a negligible effect on the results for large times or velocity gradients.

It is hard to make comparisons between the present results and a steady state pipe flow or boundary layer inasmuch as a^* contains time. However, we can make a rough estimate of the order of magnitude of a^* for a steady state case as follows. From the turbulent energy spectra in [9], Fig. 7 $\kappa^*_{\text{average}} \sim 1$. Then an average length, $1/\kappa_{\text{average}} = L$, associated with the turbulence is $[\nu(t - t_0)]^{1/2}$. Let δ be the radius of the pipe or the thickness of the boundary layer and U be a characteristic mean velocity. Letting $t - t_0 \sim L^2/\nu$ (see above), $\mathrm{d}U_1/\mathrm{d}x_2 \sim U/\delta$, and $L \sim 0{\cdot}3\delta$, a^* will be on the order of $0{\cdot}1\, U\delta/\nu$. Thus for values of mean flow Reynolds numbers usually obtained in the turbulent flows, ϵ_h/ϵ, according to Fig. 7, will probably be close to 1 for gases and liquids. For liquid metals ϵ_h/ϵ may be less than 1, in qualitative agreement with those analyses which use a modified mixing-length theory to account for heat conduction to or from an eddy as it moves transversely in a mean temperature gradient [13], [14]. In making the above comparisons, it should, of course, be remembered that the present calculations are for an idealized case which has only a partial correspondence to a passage or boundary layer. A discussion of possible differences between the two cases is given in reference [15].

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Résumé—A partir des équations de l'énergie et de Navier–Stokes, on obtient des équations pour les corrélations entre températures et vitesses en deux points dans un champ turbulent homogène. Bien que la vitesse moyenne soit uniforme et qu'il existe des gradients de température dans le champ, la turbulence décroit en fonction du temps. On obtient les solutions en mettant les équations sous forme spectrale, en prenant leur transformées de Fourier et en supposant que la turbulence est suffisamment faible pour que les corrélations triples soient négligeables devant les corrélations doubles. Les spectres de la transmission de chaleur turbulente et du carré moyen des fluctuations de température sont calculés en fonction du gradient de vitesse sans dimensions. On calcule également le rapport de la diffusivité thermique turbulente au transport de quantité de mouvement. On montre que le rapport de diffusivité turbulente tend vers l'unité aux gradients de vitesses élevés, quel que soit le nombre de Prandtl. Toutefois, il tend plus rapidement vers 1 pour les nombres de Prandtl voisins de l'unité.

Zusammenfassung—Zur Korrelation von Geschwindigkeiten und Temperaturen an zwei Punkten eines homogenen turbulenten Feldes wurden Zusammenhänge aus den Navier–Stokes-Gleichungen und den Energiegleichungen ermittelt. Trotz einheitlicher mittlerer Geschwindigkeits- und Temperaturgradienten im Feld nimmt die Turbulenz mit der Zeit ab. Lösungen wurden durch Umschreiben der Gleichungen in Spektralform mit Hilfe ihrer Fouriertransformationen erhalten und der Annahme, dass die Turbulenz für Dreifachkorrelationen gegenüber Doppelkorrelationen vernachlässigbar gering ist. Die Spektra des turbulenten Wärmeüberganges und der mittleren quadratischen Temperaturschwankung wurden als Funktion dimensionsloser Geschwindigkeitsgradienten berechnet. Das Verhältnis von turbulentem Energie- zu Impulsaustausch liess sich ebenfalls ermitteln. Es wird gezeigt, dass sich dieses Austauschverhältnis beigrossen Geschwindigkeitsgradienten unabhängig von der Grösse der Prandtlzahl dem Wert Eins nähert, die Annäherungsgeschwindigkeit an 1 aber für Prandtlzahlen in der Grössenordnung 1 am grössten ist.

Аннотация—Из уравнений Навье-Стокса и переноса энергии выведены уравнения для корреляции значений скорости и температуры в двух точках однородного турбулентного поля. Несмотря на наличие в поле постоянных средних градиентов скорости и температуры, турбулентность в нем затухает с течением времени. Решения получены путём сведения уравнений к спектральному виду, использовав преобразования Фурье и предложив, что турбулентность достаточно слаба, чтобы пренебрегать корреляциями между тремя величинами и применять корреляции между двумя величинами. Спектры турбулентного теплообмена и среднеквадратичного колебания температуры рассчитаны как функция безразмерного градиента скорости. Также получено отношение коэффициентов диффузии вихря при переносе тепла и переносе импульса. Показано, что при больших градиентах скорости без учёта влияния критерия Прандтля значение отношения коэффициентов диффузии вихря приближается к единице, а при числах Прандтля порядка I это приближение происходит в кратчайший промежуток времени.